Acceleration of particles by Janis-Newman-Winicour singularities

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We examine here the acceleration of particles and high energy collisions in the the Janis-Newman-Winicour (JNW) spacetime, which is an extension of the Schwarzschild geometry when a massless scalar field is included. We show that while the center of mass energy of collisions of particles near the event horizon of a blackhole is not significantly larger than the rest mass of the interacting particles, in an analogous situation, it could be arbitrarily large in the JNW spacetime near the naked singularity. The high energy collisions are seen to be generic in the presence of a photon sphere in the JNW spacetime, whereas an extreme fine-tuning of the angular momentum of the colliding particles is required when the photon sphere is absent. The center of mass energy of collision near the singularity grows slowly for small and extremely large deviations from the Schwarzschild blackhole, but for intermediate strengths of the scalar field it rises moderately fast. As a possible and potentially interesting application, we point out that the presence of such high energy collisions may help the blackhole configurations to be distinguished from a naked singularity.

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I. INTRODUCTION

Various terrestrial particles accelerators collide particles at the center of mass energies upto TeV scale which is significantly below the Planck scale that corresponds to the quantum gravity regime. Clearly, the physics in a very large energy range remains unexplored today. In this connection, an intriguing possibility would be to make use of various naturally occurring processes in the vicinity of various astrophysical compact massive objects where gravity would be very strong.

The blackholes are expected to provide such an environment. With this motivation it was investigated whether it is possible to have ultrahigh energy collisions in the vicinity of the event horizon of the blackholes. Various compact very massive objects that occur in the universe, for instance at the center of most of the galaxies, are assumed to be Kerr blackholes which are characterized by their spin a and mass M. In the geometrical units (where $c = G = 1$), The event horizon of such a blackhole is located at $r = b(a) = M\left(1 + \sqrt{1 - a^2}\right)$ in the Boyer-Lindquist coordinates. The center of mass energy of collision between two particles interacting near the horizon released from infinity at rest was investigated. When $a = 0$ the blackhole is a Schwarzschild blackhole and the maximum center of mass energy that can be achieved in the collision at $r = b(a = 0) = 2M$ was shown to be finite [1]. The effect of adding rotation to the blackhole was investigated in [2]. It was shown that nothing interesting happens for small values of spin. However, interestingly for large spins close to the extremality $a = 1$, it can be shown that the center of mass energy of collision at $r = b(a = 1) = M$ is arbitrarily large. Various aspects of this process in Kerr and other blackhole geometries were investigated in [3],[4]. We extended this process to superspinning objects with spin larger than unity containing a naked singularity [5],[6].

Here we take a different approach to crank up the center of mass energy of collision. Instead of spinning up a blackhole we "charge it up" with a static massless scalar field. The two parameter family solution of Einstein equations in this case is given by [9]. One parameter is mass M and the second parameter is q, the scalar charge which is a measure of strength of the scalar field as we discuss in the next section. We investigate the collision at $r = b(q) = 2\sqrt{M^2 + q^2}$. When $q = 0$, the JNW metric reduces to Schwarzschild metric and the center of mass energy of collision at $r = b(q) = 2M$ is small. We investigate here the effect of invoking a scalar field on the center of mass energy of collision at $r = b(q) = 2\sqrt{M^2 + q^2}$. We show that there is a divergence of center of mass energy of collision. However, the divergence is very slow for small and large values of q and it is moderately fast for intermediate values of q as $r = b$ is approached.

The JNW spacetime [9], is an extension of the Schwarzschild geometry when a massless scalar field is included instead of being empty. Naively speaking, the blackhole event horizon then deforms into a naked singularity in the JNW spacetime, when any smallest non-zero value q of the scalar field is included. This metric was studied from the perspective of gravitational lensing near a naked singularity [7]. If naked singularities which are hypothetical astrophysical objects occur in nature, an important problem would be how to distinguish them from the blackholes. Both these entities which occur generically in gravitational collapse of massive matter clouds within the framework of the Einstein gravity [8], present ultra-strong-gravity regimes which would be clearly of much interest to study and explore the physical effects of very strong gravity fields.

The JNW solution which always contains a naked singularity has been studied extensively from the perspective of gravitational lensing and distinguishing it from

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blackholes [7]. It was shown that for certain range of parameters, in the presence of a photon sphere, the gravitational lensing will be qualitatively similar to that by the Schwarzschild blackhole and it will not be possible to observationally distinguish the blackhole from a naked singularity configuration. But for the remaining range of the parameters in the absence of the photon sphere, the observational signatures of the two in terms of lensing will be quite different, allowing us to differentiate between the two.

We show here that it would be possible to have ultrahigh energy collisions generically in the presence of the photon sphere in the case of a JNW naked singularity, whereas in case of the Schwarzschild blackhole such collisions do not take place. Conversion of dark matter and ordinary matter particles like protons into particles with smaller mass, such as electrons, would be efficient at large center of mass energy of collisions. The flux of outgoing particles produced in collisions thus is expected to be larger in the JNW spacetime as compared to the Schwarzschild blackhole. We argue that this could lead to the observational signature of the JNW naked singularities in the presence of the photon sphere, as distinct from the blackholes.

II. THE SCHWARZSCHILD AND JNW METRICS

Schwarzschild metric is the unique solution to Einstein equations obtained with the assumptions of spherical symmetry, asymptotic flatness and absence of any source of the energy-momentum, *i.e.* a vacuum geometry, and the solution turns out to be static. It is one of the simplest and most widely studied exact solutions in the general theory of relativity. The Schwarzschild metric in the coordinates adapted to spherical symmetry and staticity (t, r, θ, ϕ) is given by the following expression,

$$ds^2 = -\left(1 - \frac{b}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)} dr^2 + r^2 d\Omega^2 \tag{1}$$

The spacetime has a strong curvature singularity at $r = 0$ which is covered by an event horizon located at $r = b$ and thus it represents a blackhole. It is characterized by the parameter $M = \frac{b}{2}$ which can be interpreted as the mass which is the source of the gravitational field.

In this paper we investigate the unique solution to the Einstein equations obtained with the assumption of spherical symmetry, asymptotic flatness with the static massless scalar field as a source of energy-momentum This solution was obtained by Janis, Newmann and Winicor [9] and independently by Wyman [10], and these were later shown to be identical [11].

The JNW metric is given by

$$ds^2 = -\left(1 - \frac{b}{r}\right)^{\nu} dt^2 + \frac{1}{\left(1 - \frac{b}{r}\right)^{\nu}} dr^2 + r^2 \left(1 - \frac{b}{r}\right)^{1-\nu} d\Omega^2 \tag{2}$$

whereas the scalar field here is given by

$$\phi = \frac{q}{b\sqrt{4\pi}} \ln\left(1 - \frac{b}{r}\right) \tag{3}$$

The solution contains two parameters

$$\nu = \frac{2M}{b} \tag{4}$$
$$b = 2\sqrt{M^2 + q^2}$$

Here M and q stand for the ADM mass and the 'scalar charge' respectively. When the scalar field is zero the solution should reduce to the Schwarzschild solution. This indeed happens and can be seen clearly by setting $q = 0$, *i.e.* $\nu = 1$ in which case (2) reduces to (1). The JNW solution differs from the Schwarzschild solution for nonzero values of the scalar charge. As we increase q, ν goes on decreasing and tends to zero for arbitrarily large values of the charge. The energy-momentum tensor of the scalar field is given here by

$$T^{\mu}_{\nu} = Diag\left[-\rho(r), p_1(r), p_2(r), p_2(r)\right] \tag{5}$$
$$\rho(r) = p_1(r) = -p_2(r) = \frac{b^2(1-\nu^2)\left(1 - \frac{b}{r}\right)^{\nu}}{4r^2\left(r - b\right)^2}$$

It is clear from the expression above that the JNW spacetime has a singularity at $r = b$ when $\nu < 1$. Thus the range of the radial coordinate is given by $b < r < \infty$. The singularity can be shown to be globally naked [11],[12]. The weak energy conditions are satisfied.

Thus to summarize, the JNW metric represents the two parameter family of solutions to the Einstein equations that is spherically symmetric and static. The two parameters b, ν, or alternatively M, q represent the mass and scalar charge. The Schwarzschild blackhole is the limiting case of JNW spacetime when $q = 0$, *i.e.* when $\nu = 1$. When $q \neq 0$ *i.e.* $0 < \nu < 1$, then $r = b$ is the naked singularity, whereas in the limiting case of the Schwarzschild blackhole $q = 0$ ($\nu = 1$), it represents the event horizon of the blackhole. Thus one might be tempted to dramatically assert that, introduction of the smallest possible static scalar field in the Schwarzschild blackhole could convert it into a JNW spacetime containing a naked singularity, and the event horizon at $r = b$ is then deformed into a naked singularity. However, while considering the statement above we must note that the JNW spacetime is not to be thought of as a perturbation of the Schwarzschild spacetime in the usual sense, *i.e.* the variation of initial data for Schwarzschild spacetime on any initial surface does not give rise to the JNW metric. It can be thought of as a variation of the Schwarzschild metric in the sense that we describe below.

Schwarzschild solution is obtained by solving the system of Einstein equations with the assumptions of spherical symmetry, asymptotic flatness and the absence of matter, *i.e.* a vacuum geometry. The solutions to Einstein equations obtained by relaxing one or more assumptions above are different and have different physical interpretations. If one sticks to the assumptions of spherical

symmetry and asymptotic flatness and introduces matter fields, one would obtain a wide variety of solutions. For instance, we arrive at Reissner-Nordstaröm geometry if one invokes the static electromagnetic field as a source of the energy-momentum. If we invoke the static massless scalar field we get JNW solution. In both these cases the departure from the Schwarzschild geometry is characterized by one parameter. In the case of electromagnetic field the parameter is an electric charge. By analogy in the case of scalar field, the additional parameter is referred to as the scalar charge. If the additional charge parameter is set to zero we recover the Schwarzschild geometry. However, there is a significant difference in the two cases. If the electric charge is small, the Reissner-Nordström geometry corresponds to a blackhole as for the asymptotic observers are concerned, and there is an event horizon present in the spacetime. However, the spacetime contains a locally naked singularity, visible in a region inside the Cauchy horizon which is the inner event horizon, but invisible to the observers outside the horizon. In contrast, in the case of JNW solutions the geometry corresponds to that of a globally naked singularity, rather than a blackhole, even for a smallest possible scalar charge, and there is no event horizon in the spacetime. This is the reason we investigate JNW solution in this paper as a representative of the variations of the Schwarzschild geometry that yield naked singularities rather than blackholes.

Looking from the sufficiently faraway region from the center, *i.e.* by observing the phenomenon that takes place in the weak field regime, like the motion of stars or planets, one can infer only the mass parameter for the system. Therefore the JNW solution essentially looks like Schwarzschild solution from such a perspective and one may not be able to distinguish between the two. So the compact massive dark object, that we might usually believe to be a blackhole, could turn out to be a naked singularity if the scalar field is present instead of it being vacuum. In order to tell one from the other, it would be necessary to investigate various physical phenomena that take place in the vicinity of this object, that is in the strong gravity regime, like the accretion of matter onto the object or gravitational lensing. This would shed light on the existence of the charge parameter and therefore the deviation of metric from being Schwarzschild. From this perspective, gravitational lensing by JNW singularities was studied in [7]. We study and present the analysis on the circular geodesics and accretion disks in the JNW geometry in a separate companion paper [13].

In this paper we investigate the effect of the inclusion of the massless static scalar field on the acceleration of particles and their collisions occurring at $r = b$.

III. PARTICLE ACCELERATION NEAR A SCHWARZSCHILD BLACKHOLE

In this section we describe the particle acceleration and collisions near the event horizon in the Schwarzschild spacetime. The event horizon of a blackhole is chosen to be a location for collision because it is a surface with infinite blueshift for the particles falling in from infinity into the blackhole. Thus naively one might think that when two such highly blueshifted particles collide near the horizon, the center of mass energy of collisions will be arbitrarily large. But actually that is not the case as we show below.

A. Geodesic motion

We first describe motion of the massive particles following timelike geodesics in the Schwarzschild spacetime. Consider a particle of mass m. Let $U^\mu = \left(U^t, U^r, U^\theta, U^\phi\right)$ be the four-velocity of the particle. The motion of the particle will be restricted to a plane because of the spherical symmetry of a spacetime. This plane can be chosen to be the equatorial plane $\theta = \frac{\pi}{2}$, exploiting the gauge freedom. Thus $U^\theta = 0$. The spherical symmetry and static nature of the Schwarzschild spacetime implies that it admits Killing vectors $\partial_\phi, \partial_t$. Using these Killing vectors one can show that the quantities $E = -\partial_t \cdot U$ and $L = -\partial_\phi \cdot U$ turn out to be the constants of motion along the geodesics. Here E, L are interpreted as the conserved energy and angular momentum per unit mass of the particle respectively. The quantities U^t, U^ϕ can be written in terms of E, L in the following way,

$$U^t = \frac{E}{\left(1 - \frac{b}{r}\right)}$$
$$U^\phi = \frac{L}{r^2} \tag{6}$$

Using the normalization condition $U.U = -1$, U^r can be written in the following way,

$$U^r = \pm\sqrt{E^2 - \left(1 - \frac{b}{r}\right)\left(1 + \frac{L^2}{r^2}\right)} \tag{7}$$

Here $\pm$ correspond to the motion in radially outward and inward directions respectively. This equation can also be cast in the following form

$$(U^r)^2 + V_{eff}(r) = E^2 \tag{8}$$
$$V_{eff}(r) = \left(1 - \frac{b}{r}\right)\left(1 + \frac{L^2}{r^2}\right)$$

where $V_{eff}(r)$ can be thought of as an effective potential for the motion in the radial direction.

We are interested here in a situation where particles that are nonrelativistic at infinity fall inwards under the force of gravity and interact with each other with large

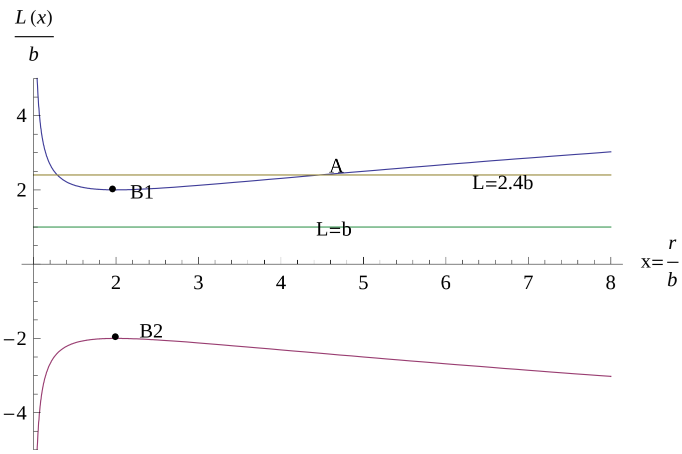

FIG. 1: The angular momentum required for a particle to turn back at radius r, namely $\frac{L}{b}$ is plotted against $x = \frac{r}{b}$. There are two branches of the graph, namely the positive and negative branches, which admit minimum and maximum at $B1, B2$ respectively. The extrema occur at $r = 2b$ and the angular momenta take extremal values $\pm 2b$. An infalling particle with angular momentum $L = b$ never turns back and reaches the event horizon, whereas the ingoing particle with an angular momentum $L = 2.4b$ which is outside this range gets reflected back at point A with the radial coordinate $r > 2b$.

center of mass energy of collision. The particles that are non-relativistic at infinity can be thought to be at rest for all practical purposes. Then $U^r \to 0$ as $r \to \infty$ implies from (7),(9) that the conserved energy of the particles must be $E = 1$. We impose this condition to ensure that if particles undergo ultrarelativistic collision, the large center of mass energy of collision can be attributed purely as an effect of infall under the gravity.

As we have mentioned before, intuitively the location of the collision must be close to the event horizon as it is a surface with infinite blueshift for the ingoing particles. Thus we must ensure that the particles that fall from rest at infinity manage to reach the event horizon and that they do not turn back well before the same is reached. This can be done in a following way.

Consider a particle following geodesic motion that turns back at r. This implies that $U^r = 0$ or $V_{eff}(r) = E^2 = 1$. From (7),(9), we get the expression of the angular momentum $L(r)$, and such a particle is required to have

$$L^2(r) = \frac{r^2 b}{r-b} \tag{9}$$
$$L(r) = \pm\sqrt{\frac{r^2 b}{r-b}}$$

The function $L(r)$ is plotted in Fig1. There are two branches of the graph corresponding to the positive and negative values of the angular momenta. The positive branch admits minimum, whereas the negative branch admits maximum at $r = 2b$. The extremal values are $-2b, 2b$. If the angular momentum of the particle is in the range $L \in (-2b, 2b)$, it never turns back and reaches the horizon. But if the angular momentum is outside this range *i.e.* when either $L > 2$ or $L < -2$, an ingoing particle would turn back at the radial value $r > 2$. A particle with angular momentum $L = \pm 2b$ would asymptotically approach $r = 2b$. This follows from the fact that for such a particle $V_{eff}(r = 2) = V'_{eff}(r = 2) = 0$, where the prime denotes the derivative with respect to r. Thus for particles infalling from infinity at rest to participate in the collisions at the event horizon of the Schwarzschild blackhole, their angular momentum must be in the range mentioned as above.

B. Collisions and Center of Mass Energy

The center of mass energy of collision [1],[2] between two identical particles each with mass m and with velocities U_1, U_2 is given by

$$E^2_{cm} = 2m^2\left(1 - g_{\mu\nu}U_1^\mu U_2^\nu\right) \tag{10}$$

The calculation of center of mass energy of collision essentially involves the computation of the inner product of the velocities of two particles.

We consider two particles each with the conserved energy $E = 1$ and angular momenta L_1, L_2 satisfying the condition $-2b < L_1, L_2 < 2b$. These particles are released from infinity at rest and manage to reach the event horizon to participate in the collision.

The center of mass energy of collision between these two particles at any given value of r can be computed using (6),(7),(10) and is given by,

$$\frac{E^2_{cm}}{2m^2} = 1 + \frac{1}{(1-\frac{b}{r})} - \frac{1}{(1-\frac{b}{r})}\sqrt{1 - \left(1 - \frac{b}{r}\right)\left(1 + \frac{L_1^2}{r^2}\right)} \sqrt{1 - \left(1 - \frac{b}{r}\right)\left(1 + \frac{L_2^2}{r^2}\right)} - \frac{L_1 L_2}{r^2} \tag{11}$$

We consider a collision close to the event horizon $r = b$. The second term under each of the square root in the expression above is much smaller than the first term. Taylor expanding the square root and keeping terms upto the first order we obtain the center of mass energy of collision close to the horizon as given by,

$$E^2_{cm} = 2m^2\left(2 + \frac{(L_1 - L_2)^2}{2r^2}\right) \tag{12}$$

The center of mass energy of collision between the two particles is maximum when angular momenta are opposite in sign and take extreme values, *i.e.* $L_1 = 2b, L_2 = -2b$.

$$E^2_{cm,max} = 20m^2 \tag{13}$$

Thus the center of mass energy of collision between two particles at horizon $r = b$ for a Schwarzschild blackhole

is not significantly large as compared to their rest mass. This result can be intitutively understood in the following way. Although the two particles are individually highly blue-shifted as they reach the horizon, they move almost parallel to each other. Their relative velocity is small. Therefore the center of mass energy of collision is small.

There are a number of ways to crank up and raise the center of mass energy of collision. For instance one could spin up or charge up the blackhole. Nothing interesting happens really even in the case of the Kerr or Reisnner-Nordstrom blackholes unless one reaches the extremal limit. However, the center of mass energy of collision between the particles is shown to be divergent near the event horizon of extremal or near extremal blackholes [2][3], provided certain fine-tuning assumptions are obeyed. We also extended the particle acceleration mechanism to near extremal naked singularities [5],[6].

We note that one of the main drawbacks of the particle acceleration process by near extremal blackhole has been that the extreme finetuning of the angular momentum of one the particles was required for the divergence of the center of mass energy of collision.

In this paper we suggest a different way to crank up the center of mass energy of collision by "charging up" the Schwarzschild blackhole with a static spherically symmetric scalar field. By doing that we obtain the JNW solution. We study the particle acceleration in the JNW metric in the next section.

IV. PARTICLE ACCELERATION IN JNW SPACETIMES

In the previous section we investigated the collision of the particles near the event horizon of the blackhole $r = b$, and showed that the center of mass energy of collision between the particles was not significantly larger compared to the rest mass. In this section we study the collision between the particles at $r = b$ in the JNW spacetime obtained from the Schwarzschild geometry by inclusion of the massless scalar field. We show that the center of mass energy of collision in JNW spacetime can be arbitrarily large. Thus inclusion of the scalar field changes the phenomenon of particle acceleration significantly. We also investigate here the genericity of the ultrahigh collision process in terms of the allowed values of the geodesic parameters associated with the particles participating in the ultrahigh energy collisions.

A. Geodesic motion in JNW spacetime

We describe here the motion of the massive particles following the timelike geodesics in the JNW spacetime. Consider a particle of mass m and four velocity $U^\mu = \left(U^t, U^r, U^\theta, U^\phi\right)$. As in the case of Schwarzschild blackhole, this particle moves in a plane which can be taken to be an equatorial plane, and thus $U^\theta = 0$. Its motion can be described in terms of the constants of motion $E = -\partial_t.U$ and $L = \partial_\phi.U$, namely the conserved energy and angular momentum per unit mass of the particle. Using the constants of motion and (2), U^t, U^ϕ can be written as,

$$U^t = \frac{E}{\left(1-\frac{b}{r}\right)^\nu}$$
$$U^\phi = \frac{L}{r^2\left(1-\frac{b}{r}\right)^{1-\nu}} \tag{14}$$

Using the normalization condition for the velocity, namely $U.U = -1$, U^r can be written as

$$U^r = \pm\sqrt{E^2 - \left(1-\frac{b}{r}\right)^\nu\left(1+\frac{L^2}{r^2\left(1-\frac{b}{r}\right)^{1-\nu}}\right)} \tag{15}$$

Here $\pm$ correspond to the motion in radially outward and inward directions respectively. This equation can also be written in the following form

$$(U^r)^2 + V_{eff}(r) = E^2 \tag{16}$$
$$V_{eff}(r) = \left(1-\frac{b}{r}\right)^\nu\left(1+\frac{L^2}{r^2\left(1-\frac{b}{r}\right)^{1-\nu}}\right)$$

where $V_{eff}(r)$ can be thought of as an effective potential for the motion in the radial direction.

We set $E = 1$ since we consider particles that are released from infinity at rest and which fall freely under the gravity. Here "infinity" corresponds to $r \to \infty$ subject to $\nu \in (0, 1)$. Since we are interested in the collisions of the particles at $r = b$, we must ensure that the particles are not reflected back before they reach there.

Consider a particle that turns back at a given radial coordinate r. We must have $U^r(r) = 0$ or $V_{eff}(r) = 1$. It follows from (15),(17) that if the particle were to turn back from r it must have an angular momentum which is given by,

$$L^2 = \left[\left(1-\frac{b}{r}\right)^{1-2\nu} - \left(1-\frac{b}{r}\right)^{1-\nu}\right] r^2$$
$$L = \pm\sqrt{\left[\left(1-\frac{b}{r}\right)^{1-2\nu} - \left(1-\frac{b}{r}\right)^{1-\nu}\right]} r \tag{17}$$

There are two branches of the angular momentum, namely the positive branch and negative branch.

The parameter ν ranges from the values 0 and 1. For all values of ν the power of the second term in the parenthesis $(1-\nu)$ is nonnegative whereas the power of the first term $(1-2\nu)$ can either be positive, negative or zero depending on whether $\nu < \frac{1}{2}$,$\nu > \frac{1}{2}$ or $\nu = \frac{1}{2}$. Thus as we approach $r = b$ the first term can either go to zero, blow up or remain finite. We analyze these three cases separately here. Interestingly, as we discuss in the next

section, $\nu = \frac{1}{2}$ also marks the disappearance of the photon sphere in the spacetime. For $\nu < \frac{1}{2}$ the photon sphere is absent, whereas for $\nu > \frac{1}{2}$, photon sphere is present in the spacetime.

1. Geodesics in JNW spacetime with $\nu < \frac{1}{2}$

We first discuss the case when $\nu < \frac{1}{2}$. We focus on the positive branch of the angular momentum. The discussion for the negative branch will be identical to that of the positive branch due to the mirror symmetry. For large values of r, the angular momentum required for the particle to turn back has a following behavior,

$$L \approx \sqrt{\nu r b} \tag{18}$$

On the other hand, for values of $r \to b$ we have,

$$L \to 0 \tag{19}$$

Thus for a particle to reach near $r = b$ and participate in the collision it must have an angular momentum tending to zero. Thus we need a fine-tuning of the angular momentum of both the particles to a vanishingly small value for them to reach arbitrarily close to $r = b$ and participate in the collision.

2. Geodesics in JNW spacetime with $\nu = \frac{1}{2}$

For the case $\nu = \frac{1}{2}$, angular momentum required for the particle to turn back at r is given by

$$L(r) = r\sqrt{1 - \left(1 - \frac{b}{r}\right)^{\frac{1}{2}}} \tag{20}$$

We are dealing with the positive branch here and this is plotted in Fig2.

For large values of r the angular momentum goes as

$$L(r) \approx \sqrt{\frac{br}{2}} \tag{21}$$

whereas at $r = b$ it attains a constant value

$$L(r) \approx b \tag{22}$$

It admits a local minimum at $r = 1.125$ and the minimum value of the angular momentum is given by

$$L_{min} = 0.844b \tag{23}$$

By mirror symmetry the negative branch will admit a maximum at the same value of the radial coordinate and the value of the angular momentum will be the negative of the minimum for the positive branch.

Thus the ingoing particles from infinity with angular momentum in the range

$$-0.844b < L < 0.844b \tag{24}$$

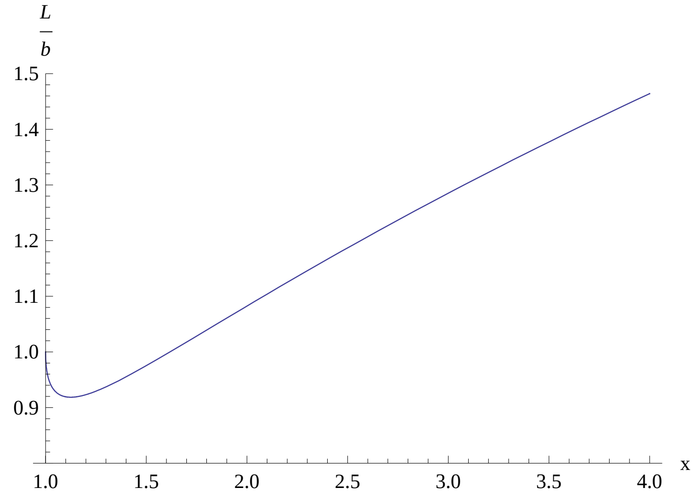

FIG. 2: The positive branch of angular momentum required for a particle to turn back at r, namely $\frac{L}{b}$ is plotted against $x = \frac{r}{b}$. At $r = b$, the angular momentum takes a finite value. It admits a minimum at $r = 0.125b$ and the minimum value is $L_{min} = 0.84b$. Particles traveling inwards would reach $r = b$ if their angular momentum is in the range $L \in (-L_{min}, L_{min})$. Otherwise they would turn back.

will be able to reach $r = b$ and participate in the collisions. Whereas particles with the angular momentum outside this range will get deflected away from $r > 1.125b$.

3. Geodesics in JNW spacetime with $\nu > \frac{1}{2}$

Finally we consider the third case where $1 > \nu > 1/2$. We analyze the positive branch of the angular momentum for the particle to turn back. For large values of r, as earlier angular momentum has a following behavior

$$L \approx \sqrt{\nu r b} \tag{25}$$

For small values of $r \to b$ we get

$$L \to \infty \tag{26}$$

It admits a minimum for an intermediate value of r. It is difficult to analyze the minimum analytically. We plot in Fig3 the angular momentum against r on x-axis and ν on y-axis.

It can be clearly seen that the radial coordinate at which the minimum is attained, as well as the minimum value of the angular momentum, increases as we increase ν from 0.5 to 1. Thus it follows that if the angular momentum of the ingoing particle is in the range

$$-0.844b < L < 0.844b \tag{27}$$

it would definitely reach $r = b$ to participate in the collision for various values of ν.

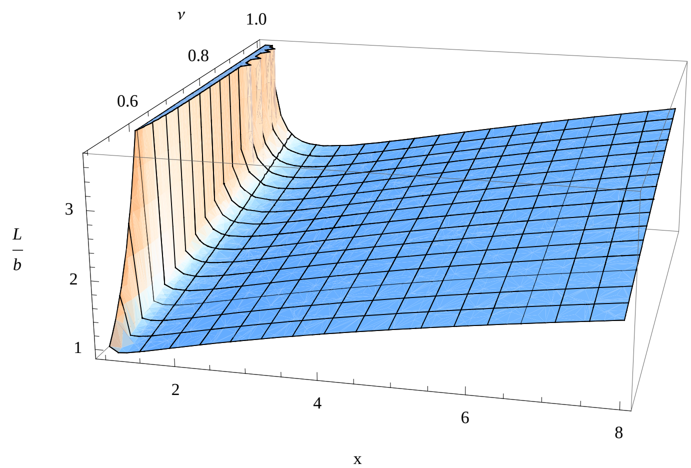

FIG. 3: The positive branch of angular momentum required for a particle to turn back at r, namely $\frac{L}{b}$ is plotted against on x-axis $x = \frac{r}{b}$ and on y-axis ν. The range of ν is from 0.5 to 1. The angular momentum blows up at $r = b$. The value of radial coordinate r where minimum of the angular momentum is attained and the minimum value of the angular momentum goes on increasing with increasing ν.

B. Collisions and Center of Mass Energy

We now describe the acceleration collision of particles in JNW spacetime. We show that unlike the Schwarzschild blackhole case it is possible to have high energy collsions at $r = b$ when the scalar field is nonzero. We also examine the genericity of the process in terms of the allowed range of the angular momentum of the colliding particles participating in the high energy collisions.

1. Case $0 < \nu < \frac{1}{2}$

We first consider the case where $\nu < \frac{1}{2}$. Two identical particles of mass m are released from infinity at rest. If they were to reach and interact in the vicinity of $r = b$ they must have a vanishingly small angular momentum. We assume that the collision taken place at r close to $r = b$ and two particles have angular momenta given by (17) $L(r) \to 0$ that would make their component of radial velocity U^r to be identically zero. We assume that one of the particles has a positive and the other has a negative angular momentum. This is because if they have the same angular momentum, they have identical velocities, their relative velocity is zero and thus the center of mass energy of collision will be $2m$.

The center of mass energy of collision between the particles, using (10),(14),(15) is given by,

$$E_{cm}^2 = 2m^2 \left(1 - g_{\mu\nu} U_1^\mu U_2^\nu\right) \tag{28}$$

$$= 2m^2 \left(1 + \frac{1}{\left(1 - \frac{b}{r}\right)^\nu} + \frac{L(r)^2}{\left(1 - \frac{b}{r}\right)^{1-\nu} r^2}\right)$$

Using (17) we get

$$E_{cm}^2 = \frac{4m^2}{\left(1 - \frac{b}{r}\right)^\nu} \tag{29}$$

$$\lim_{r \to b} E_{cm}^2 \to \infty$$

It follows that the center of mass energy of collision blows up as we approach $r = b$.

Fig4 shows the variation of center of mass energy of collision as a function of radial coordinate and ν. For small values of ν the divergence is slow and one has to go very close to $r = b$ to obtain sufficiently large center of mass energies. But for $\nu \approx 0.5$ we get large center of mass energies at moderate distance from $r = b$.

One limitation of the particle acceleration process for this range of parameter values for ν is that the angular momentum must be arbitrarily close to being zero. Thus the fine-tuning of angular momentum is necessary. Values of parameters in this range correspond to large deviation from the Schwarzschild geometry. It turns out that it is possible to have high energy collisions, however, this process is very much fine-tuned.

2. Case $\frac{1}{2} \leq \nu < 1$

We now study the particle collisions for the remaining values of the parameter ν, namely $\frac{1}{2} \leq \nu < 1$. We present a combined analysis of cases $\nu = \frac{1}{2}$ and $\frac{1}{2} < \nu < 1$ which we had analyzed previously because there was a qualitative difference in the behavior of the angular momentum function $L(r)$ near $r = b$. However, while calculating the center of mass energy of collision they can be analyzed together.

We again consider two particles released from infinity at rest with the angular momenta in the range given by(24) so that they reach $r = b$ and participate in the collision. The center of mass energy of collision between these two particles (10),(14),(15) is given by,

$$E_{cm}^2 = 2m^2 \left(1 - g_{\mu\nu} U_1^\mu U_2^\nu\right) \tag{30}$$

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{1}{\left(1 - \frac{b}{r}\right)^\nu} - \frac{L_1 L_2}{r^2 \left(1 - \frac{b}{r}\right)^{1-\nu}} - \frac{1}{\left(1 - \frac{b}{r}\right)^\nu} \sqrt{1 - \left(1 - \frac{b}{r}\right)^\nu \left(1 + \frac{L_1^2}{r^2 \left(1 - \frac{b}{r}\right)^{1-\nu}}\right)} \sqrt{1 - \left(1 - \frac{b}{r}\right)^\nu \left(1 + \frac{L_1^2}{r^2 \left(1 - \frac{b}{r}\right)^{1-\nu}}\right)} \tag{31}$$

The second term under the square root close to $r = b$ is negligible as compared to the first term. This follows

from the fact that $\nu > \frac{1}{2}$. Thus Taylor expanding the square root and neglecting the terms beyond first order we get,

$$E_{cm}^2 = 2m^2 \left(2 + \frac{1}{2} \frac{(L_1 - L_2)^2}{2\left(1 - \frac{b}{r}\right)^{1-\nu} r^2} - \frac{L_1^2 L_2^2 \left(1 - \frac{b}{r}\right)^{3\nu - 2}}{r^4} \right) \quad (32)$$

$$\lim_{r \to b} E_{cm}^2 \to \infty$$

The third term in the above is important only when $\nu \in (\frac{1}{2}, \frac{2}{3})$, and it can be neglected for the remaining values of the parameter ν. Even in this range, the second term dominates the first term since $(1 - \nu) > (2 - 3\nu)$.

The center of mass energy diverges as $r = b$ is approached. The divergence is very slow for $\nu \approx 1$, whereas it is moderately fast for $\nu \approx \frac{1}{2}$.

The process of particle acceleration and high energy collision is generic for this range of values of parameters since the angular momenta of the particles participating in the collision lie in a finite range, unlike those taking a single fine-tuned value.

V. OBSERVATIONALLY DISTINGUISHING THE SCHWARZSCHILD BLACKHOLE FROM JNW NAKED SINGULARITY

Gravitational lensing has been suggested as a way of distinguishing the blackholes from JNW naked singularities that occur if the scalar field is present in the spacetime [7]. The gravitational lensing phenomenon, however, depends crucially on whether or not a photon sphere is present in the spacetime. In the presence of the photon sphere gravitational lensing by the JNW spacetime turns out to be qualitatively similar to that by the Schwarzschild blackhole. However, in the absence of the photon sphere the gravitational lensing in the JNW geometry is qualitatively different from the blackhole case.

The photon sphere is present in JNW spacetime when $\frac{1}{2} < \nu < 1$, whereas it is absent when $0 < \nu < \frac{1}{2}$. Thus gravitational lensing as discussed in [7],will be a useful observational technique to distinguish blackholes from JNW naked singularity when $0 < \nu < \frac{1}{2}$, and it will be essentially ineffective when the scalar field strengths lie in the range $\frac{1}{2} < \nu < 1$.

However, as we have now shown in the previous section, it is possible to have high energy particle collisions around the JNW naked singularity for the range of values $\frac{1}{2} < \nu < 1$. On the other hand, the particle collisions around the blackhole will be low center of mass energy collisions.

What this means is, a naked singularity can have different possible manifestations in terms of the physical effects it may generate. The gravitational lensing could be one such effect which may help us differentiate the blackholes from a naked singularity which is a hypothetical astrophysical object, when it occurs in nature. But when that fails, for example, in the case of presence of a photon sphere in the case of the JNW solution, then there could be other physical effects such as the acceleration and high energy collision of the particles, which could give characteristic signatures to differentiate the naked singularity from a blackhole.

The cross-sections for various particle physics processes depend on the center of mass energy of collisions between the particles. It increases typically with the center of mass energy. Thus there would be a characteristic difference between the collisions that occur around the blackhole and the JNW naked singularities. Dark matter as well as ordinary matter particles like protons would be falling in towards the central massive object due to strong gravity, and hence must undergo collisions. In the case of blackholes the collisions will be small center of mass energy collisions, whereas in case of JNW naked singularities the center of mass energy collisions will be very large. Since the cross section for various particle physics processes in general would be large at the large center of mass energy of collisions, therefore the flux of the outgoing particles like electrons, neutrinos and so on which have been produced in the collisions could be expected to be quite large in the case of JNW naked singularities as compared to their blackhole counterparts.

Such a scheme can then be possibly used to observationally distinguish blackholes and the JNW naked singularities with the parameter range $\frac{1}{2} < \nu < 1$, where the gravitational lensing proves ineffective for the purpose of distinction of the two due to the presence of the photon sphere.

VI. CONCLUDING REMARKS

In this paper we studied the effect of inclusion of a scalar field onto the center of mass energy of collision between the particles. The main purpose here was to show that it is possible to have ultrahigh energy collisions in the JNW spacetime in the presence of a static massless scalar field around naked singularities, while the high energy collisions are absent in the Schwarzschild blackhole case when the scalar field is zero.

We also speculated that the high energy collisions could allow us to set up a scheme to observationally distinguish the Schwarzschild blackhole from the JNW naked singularities which are surrounded by the photon sphere, in the case when the gravitational lensing is ineffective for such a purpose. It is beyond the scope of this paper to rigorously demonstrate it here and we plan to report that work elsewhere.

Also, in the future work we would like to study the process of infall of dark matter particles and also ordinary particles like protons onto the galactic central dark object, modeled as a blackhole and as a JNW spacetime with a photon sphere surrounding the naked singularity. A good understanding of the density of dark matter and ordinary matter particles in the surroundings of these objects will have to be obtained and studied. We intend

to study the collisions and investigate various particle physics processes leading to the annihilation of dark and ordinary matter particles into other particles like neutrinos and such other by-products. Finally, one would like to calculate the flux of the particles produced in the collisions at the large distances for JNW naked singularities, and compare it with the corresponding flux in the blackhole case.

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